Current and Resistance

Current

- In our previous discussion all of the charges that were encountered were stationary, not moving.
- If the charges have a velocity relative to some reference frame then we have a current of charge.

Current

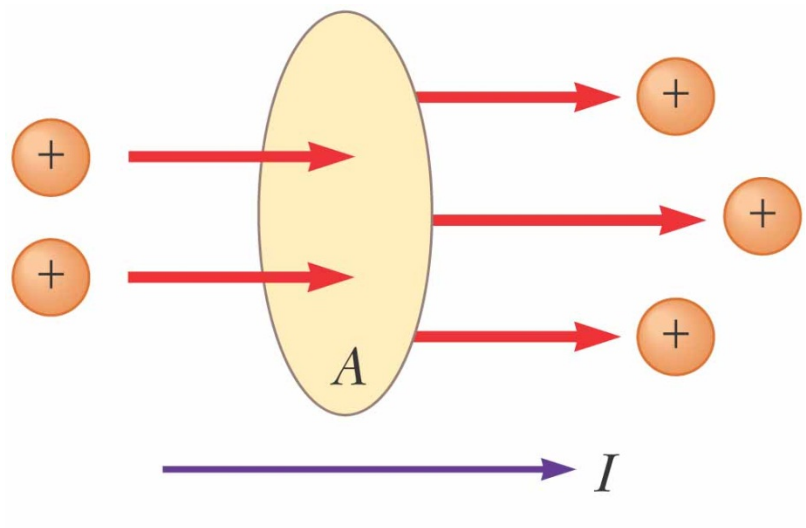
- Definition of current:

Note

- The current may or may not be a function of time.
- If a battery is initially hooked up to a loop of wire there is a potential difference between on end of the wire and the other, therefore, the charges in the wire will begin to move.
- Once equilibrium is reached the amount of charge passing a given point will be constant.
- However, before equilibrium the current will be increasing and therefore it will be a function of time.

Average Electric Current

- Assume charges are moving perpendicular to a surface of area *A*
- If ⊗*Q* is the amount of charge that passes through *A* in time ⊗*t*, then the average current is



$$I_{av} = \frac{\Delta Q}{\Delta t}$$

Instantaneous Electric Current

- If the rate at which the charge flows varies with time, the instantaneous current, I, can be found

Direction of Current

- The charges passing through the area could be positive or negative or both
- It is conventional to assign to the current the same direction as the flow of positive charges
- The direction of current flow is opposite the direction of the flow of electrons
- It is common to refer to any moving charge as a *charge carrier*

Current Density

We can define the current density as the current per unit area through a surface.

The current can now be expressed as:

Current Density

- Here d**A** is a vector that is perpendicular to the differential surface area dA.
- If the current is uniform across the surface and parallel to d**A** then we can write:

Example

- The Los Alamos Meson Physics Facility accelerator has a maximum average proton current of 1.0 mA at an energy of 800 Mev.

Example cont.

- a) How many protons per second strike a target exposed to this beam if the beam is of circular cross section with a diameter of 5 mm?
- b) What is the current density?

Solution

- a) The number of protons per second is:

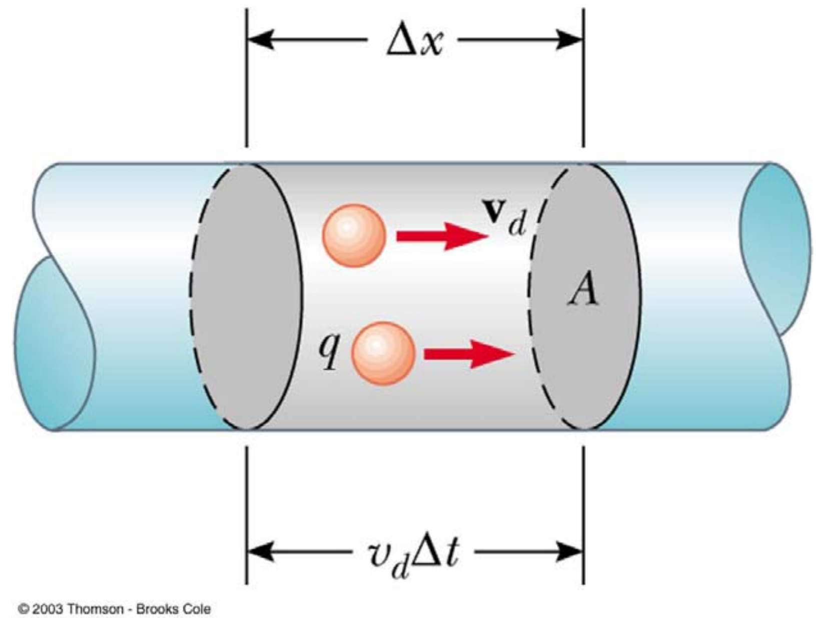
- Here *n* is the number of protons per second and *e* is the charge of the proton.

Solution cont.

- b) The magnitude of the current density for this problem is just the current divided by the cross sectional area.

Drift Speed

- When a current is established in a circuit the electrons drift through the circuit with a speed that is related to the applied electric field.
- To determine the drift speed, imagine a section of wire of length L and cross sectional area A with number, n equal to the number of electrons per volume.



Drift Speed

- If the electrons all have the same speed then the time for them to move across the length L of the wire is:

Drift Speed

- The current is then:

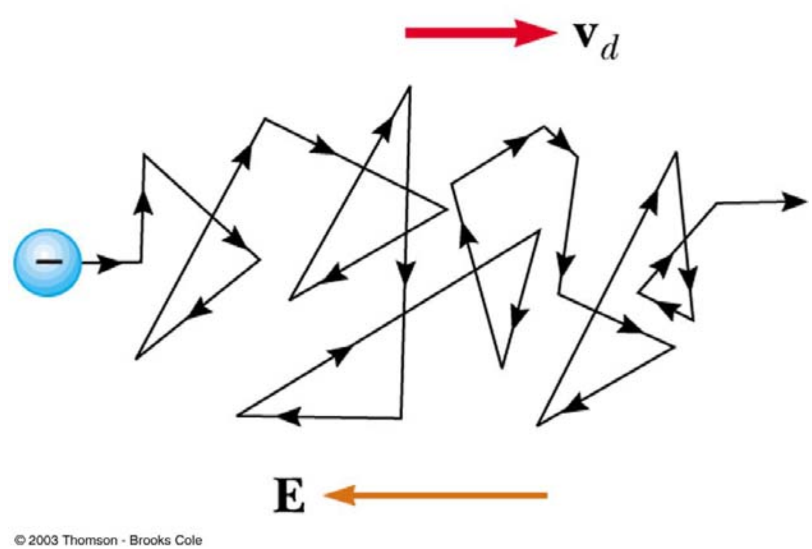
Drift Velocity

- The magnitude of the drift velocity can now be expressed as:

Then the current density is:

Charge Carrier Motion in a Conductor

- The zigzag black line represents the motion of a charge carrier in a conductor
  - The net drift speed is small
- The sharp changes in direction are due to collisions
- The net motion of electrons is opposite the direction of the electric field



Example Nerve Conduction

- Suppose a large nerve fiber running to a muscle in the leg has a diameter of 0.25 mm.
- When the current in the nerve is 0.05 mA, the drift velocity is 2.0×10^{-6} m/s.
- If we model this problem by assuming free electrons are the charge carriers, what is the density of the free electrons in the nerve fiber?

Solution

- We first calculate the cross-sectional area of the nerve fiber.

- The current density is then:

Solution cont.

- We can now calculate the density of the free electrons.

Resistance

- The resistance of a circuit is defined as the potential drop across the circuit divided by the current that pass through the circuit.
- The unit for resistance is the ohm $\Omega = 1V/A$.

Resistivity

- The resistivity of a material is defined as:
- The unit for resistivity is the ohm-meter.
- The resistance is a property of the entire object while the resistivity is a property of the material with which the object is made.

Resistance

- The relationship between resistance and resistivity is:

Resistivity and Conductivity

- The electric field can now be written in terms of the current and resistivity of the circuit.
- The conductivity of a material is the reciprocal of the resistivity.

Ohm’s Law

- Ohm's law states that the current through a device is directly proportional to the potential difference applied to the device.

- Note: Not all circuits obey Ohm's law.
- If the resistance is a function of the applied potential difference then the circuit will not obey Ohm's law.

Ohm’s Law cont.

- Ohm's law can be expressed by the following vector equation:

- An equivalent scalar equation for Ohm's law is given by:

Power in Electric Circuits

- By definition power is given as:

- Here P is power and U is the potential energy.

- The electric potential energy is given by:

Power in Electric Circuits

- We can now obtain the power of a circuit by differentiating the energy with respect to time.

Power in Electric Circuits

- If the potential difference is a constant with the time then the power can be expressed as:

Other Forms of Power

- If we use Ohm's Law we can express the power as:

- The power of the circuit is the power dissipated by the resistance of the circuit.

Example

- Nikita, one of Section One’s top operatives, finds herself in a life-threatening situation. Red Cell has captured her and placed her in a containment cell with a large steel, electric locking, door. Nikita’s only chance to escape is to short-circuit the switch on the door from the inside.

Example cont.

- The switch has a fuse that will blow once the current exceeds 5.0 amps for more than 1.5s.
- Nikita has smuggled a small electrical device, given to her by Walter, into the cell.
- The device has a power rating of 25 W.

Example cont.

- a) What must the voltage of the device be in order to short-out the lock on the door?

- b) If the device has 50 J of energy stored in it, can Nikita open the door with this device?

Solution part a

- a) We can use the power equation to determine the minimum voltage needed to blow the fuse.

Solution part b

- b) The energy needed to blow the fuse can be determine by the following:

Resistance as a Function of Temperature

- We can express the temperature dependence of resistance in terms of the the temperature coefficient of resistivity.

Resistance and Temperature

- We can solve this linear-first-order ordinary differential equation by using separation of variables method.

Resistance and Temperature

- If we integrate and solve for the resistivity we get the resistivity as a function of temperature.
- Note: as the temperature increase so does the resistivity.